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M.A./M.Sc. THIRD SEMESTER EXAMINATION, JANUARY 2015

SECOND YEAR

MATHEMATICS

Date : 19/01/2015

Time : 11 am – 1 pm

Paper : XIV [Mechanics]

Full Marks : 50

[*Answer Question No. 1 and any five from the rest*]

1. Attempt **any two** : [2×5]
   a) Derive equations of motion of a particle in spherical polar coordinates using Lagrange's equations of motion.
   b) If $F_2(q_r, P_r, t)$ be the generating function for the Canonical transformation from the old set (q_r, p_r) to the new set of Coordinates (Q_r, P_r), show that the new set of coordinates (Q_r, P_r) and the new Hamiltonian $\bar{H}(Q_r, P_r, t)$ can be derived from the equations $p_r = \dfrac{\partial F_2(q_r, P_r, t)}{\partial q_r}$, $Q_r = \dfrac{\partial F_2(q_r, P_r, t)}{\partial P_r}$ and $\bar{H} = H + \dfrac{\partial F_2(q_r, P_r, t)}{\partial t}$.
   c) Derive Hamilton-Jacobi equation in the form $H\left(q_1, q_2, \ldots q_n, \dfrac{\partial S}{\partial q_1}, \dfrac{\partial S}{\partial q_2}, \ldots \dfrac{\partial S}{\partial q_n}, t\right) + \dfrac{\partial S}{\partial t} = 0$. Explain the physical significance of S?

2. Derive equations of motion of a particle of mass m referred to the axes on the revolving earth, considering λ be the geographical latitude of the place and (mX, mY, mZ) be the components of applied force in addition to the force of attraction due to gravity. [8]

3. A bead of mass m slides on a smooth uniform circular wire of radius r = a which is rotating with constant angular velocity ω about a fixed vertical diameter. Set up the Lagrangian and find the equation of motion of the bead.
   If the bead is released with no vertical velocity from a point on the level of the centre of the circular wire, show that it will not reach the lowest point if $\omega > \sqrt{\dfrac{2g}{a}}$. [8]

4. What do you mean by Ignorable coordinates? Explain Routh's procedure to solve problems involving ignorable coordinates. [8]

5. A particle of mass m is constrained to move on the surface of a cylinder whose equation is $x^2 + y^2 = R^2$. The particle is subjected to a force $F = -\mu r$ directed towards the origin, where r is the distance of the particle from the origin. Use Hamilton's equations of motion to show that the motion of the particle in z-direction is simple harmonic. [8]

6. O and O' are two fixed points in space not in the same vertical line, O being at higher level than O'. Find the form of the plane curve joining O and O' such that the time of fall of a smooth particle from O to O' along this curve starting from rest at O and moving under gravity is minimum. [8]

7. If the transformation equations between two sets of coordinates are
   $$P = 2\left(1 + q^{1/2}\cos p\right) q^{1/2} \sin p$$
   $$Q = \log\left(1 + q^{1/2}\cos p\right)$$
   then show that the transformation is Canonical and the generating function of the transformation is $F_3(p, Q) = -(e^Q - 1)^2 \tan p$ [8]

8. Expand the dynamical Variable F(q, p) in terms of Poisson Bracket in the form

$$F = F_0 + t[F,H]_0 + \frac{t^2}{2!}\big[[F,H],H\big]_0 + \frac{t^3}{3!}\big[\big[[F,H],H\big],H\big]_0 + \ldots$$

where the subscript 0 (zero) means the values of the corresponding quantities at t = 0. Use the above result to determine the position of a particle of mass m at time t moving with constant acceleration f in x-direction starting with velocity v_0 from the position $x = x_0$ at t = 0. [8]

9. State Hamilton's Principle. Derive Hamilton's Canonical equations of motion from it. [8]

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